**Mathematical outlines**

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# Introduction of Equations of motion

Explicit:

Implicit:

Keep only , we obtain

size , the same size as

size

Keep only , we obtain . Two solutions:

**First**, one Newton, change nonlinear into linear:

**Second**, change it into convex optimization problem like:

View , , , set , then

这一块的能量还是有问题的，不知道怎么过去的，需要换种思路，要是积分才能过去？

Given all the force energy and as the force including elasticity, gravity, etc., where , . Set . To obtain the minimal value, we need to calculate gradient, such that

Therefore

Then we only need to find such that . Then we can turn to Newton method for help to calculate for the solution:

And only thing we don’t know is , for elasticity:

However,

Is this because this is linear FEM, then ? So we only need the first term?

所以实际上二阶导数并不难求，关键是求法会比较困难，可以仔细研究一下Dynamic Deformable的写法，有详细的关于不同stress tensor写法的求导方案

Then we obtain several problems:

1. What is looks like? Constitutive model
2. How to calculate exactly? Finite element method
3. How to find solutions of convex problem? Numerical methods
4. Other Tips? fixed Hessian to ensure convex

# Constitutive model

Deformation gradient

Energy density .

Strain tensor describe severity of deformation, eg: Venant-Kirchhoff , co-rotated linear model

Energy density should looks like , and energy should be

Gradient of , where , inner product of matrix

first Piola-Kirchhoff stress tensor.

Note

St. Venant-Kirchhoff model(StVK):

Co-rotated linear model:

NeoHookean:

Implicit NeoHookean

One element Hessian should be like

#四个顶点全都初始化成0

#一个element三个点 初值化为ej\*ek^T

#初始化为x,y,z维其余三个矩阵之和的复数

#初始化dF 对一个element 把张量里面的每个3\*3矩阵都乘上B

Matrix calculus

For NeoHookean , as below

and are all fourth order tensor

First write element , use two dimensions as example

Implicit method with Newton Method, again we check the motion equation:

The only thing we don’t know the nonlinear function , and solve the equation is equivalent with solving

Set

Where and are model dependent. For elasticity

Notice that if we would like to use instead of like

This is exactly like equation of , more importantly, only can make Newton Method converge

Then apply Newton Method, since newton method is to calculate , such that

When small enough, we have converged

Step 1: initial guess

Step 2: while not converge:

Then finally solution what we want

To be specific,

# Finite element method

We should use 3-dim Gauss quadrature, choose several points and affine from -1,1 to 0,1

# Numerical methods

# Other Tips